Language-based abstractions for dynamical systems

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Ordinary differential equations (ODEs) are the primary means to model dynamical systems in a wide range of natural and engineering sciences. When the complexity of the considered system is high, the number of equations required limits our capability of performing effective analysis. This has motivated a large body of research, across many disciplines, into abstraction techniques that provide smaller ODE systems preserving the original dynamics in some appropriate sense (e.g., [2, 14, 15, 5]).

Our own line of research [10, 18, 7, 9, 20, 8, 6, 13] consists of a computer science perspective to this problem, borrowing ideas from the concurrency theory community. We recast the ODE reduction problem to that of finding an appropriate equivalence relation over ODE variables, akin to classical models of computation based on labelled transition systems. We studied such "differential equivalences" for two basic intermediate languages, trading expressivity for efficiency:

i) *IDOL* (Intermediate Drift-Oriented Language) [9] covers a general class of non-linear ODEs with derivatives containing polynomials, rationals, minima/maxima, and absolute values. This is sufficient, e.g., to capture the existing ODE semantics of stochastic process algebras [13, 4]. The largest equivalences of IDOL terms are computed using a symbolic partition-refinement algorithm that exploits an encoding into a satisfiability modulo theories (SMT) problem;

ii) *Reaction networks* [6, 8], a slight generalization of chemical reaction networks, characterise ODEs with polynomial derivatives. In this case, the partition refinement is based on Paige and Tarjan's seminal proposal [16], giving an efficient algorithm that runs in polynomial time.

A tutorial-like presentation unifying the two approaches can be found in [20], while [7, 18] address the more general problem of computing all differential equivalences of a model. Our framework for ODE reduction has been implemented in the tool ERODE [10] (`http://sysma.imtlucca.it/tools/erode/`), allowing us to provide evidence of effective reductions in realistic models from the literature.

We remark that our techniques consider exact aggregations. In some cases, however, one might be interested in more permissive, approximate, notions that do not discriminate ODE variables with nearby trajectories in practice (e.g., [17, 1, 3, 11, 19, 12]). In an ongoing research we are developing approximate variants of our differential equivalences, aiming at maintaining computational tractability, and certified error bounds that do not grow fast with time.

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